The Math Dude's
Quick and Dirty Guide to Algebra

053-47216_ch00_5P.indd i 5/10/11 4:40 PM

053-47216_ch00_5P.indd ii 5/10/11 4:40 PM

The Math Dude's

Quick and Dirty Guide to Algebra

JASON MARSHALL

ST. MARTIN'S GRIFFIN NEW YORK

053-47216_ch00_5P.indd iii 5/10/11 4:40 PM

THE MATH DUDE'S QUICK AND DIRTY GUIDE TO ALGEBRA. Copyright © 2011 by Jason Marshall. All rights reserved. Printed in the United States of America. For information, address St. Martin's Press, 175 Fifth Avenue, New York, N.Y. 10010.

www.stmartins.com

ISBN 978-0-312-56956-3

First Edition: July 2011

10 9 8 7 6 5 4 3 2 1

This book is dedicated to all the "math fans" of the world. And, in particular, to the great community of *The Math Dude* podcast listeners. Without your interest and support there would be math, but there would be no *Math Dude*.

Thank you!

053-47216_ch00_5P.indd v 5/10/11 4:40 PM

053-47216_ch00_5P.indd vi 5/10/11 4:40 PM

CONTENTS

INTRODUCTION 1

What Can This Book Do for You? 1

How Should You Use This Book? 7

PROLOGUE: WHY MATH ISN'T AN AWFUL NERD 9

Basic Number Properties 10

Basic Arithmetic 11

Let the Game Begin! 13

Looking for Patterns in Numbers 17

Exponents and Perfect Squares 19

A Surprising Sequence of Numbers 20

Wrap-up 26

Final Exam 27

PART I: WHAT IS ALGEBRA, REALLY?

CHAPTER 1: TAKING ALGEBRA TO THE STREETS 33

The Secret Algebra You've Already Been Doing 34

Using Variables 38

Writing Equations 40
What's the Point of Algebra? 42
Square Roots 46
The Pythagorean Theorem 49
Why Algebra Matters in the Real World 56
Algebra in Your Backyard 62
Algebra Tutorial: How to Make a Graph 63
Challenge Problems 71
Final Exam 72

CHAPTER 2: ALGEBRA BASICS 76
What Are Variables? 77
How Do Variables Work? 82
What Are Algebraic Expressions? 86
Algebra Tutorial: The Order of Operations 88
Algebra Tutorial: Practice Problem 92
Intro to Equations 95
Halftime Recap 99
How You Should Think About Equations 106
Algebra Tutorial: How to Solve an Equation 107
Algebra Tutorial: How to Solve an Algebra Problem 123
Wrap-up 129
Final Exam 130

PART II: UNDERSTANDING ALGEBRA BETTER

CHAPTER 3: WALK THE NUMBER LINE 135
A Brief History of Numbers 136
Algebra and Decimal Numbers 141
Algebra and the Number Line 143
Absolute Values 144

Number Boot Camp 155
Linear Equations 164
Algebra Tutorial: How to Solve Single Variable Linear Equations 167
Linear Equations with Absolute Values 174
Algebra Tutorial: How to Solve Absolute Value Equations 176
Linear Inequalities 179
Algebra Tutorial: How to Solve Linear Inequalities 181
Wrap-up 193
Final Exam 194

CHAPTER 4: ARITHMETIC 2.0: MATH WITH VARIABLES, EXPONENTS, AND ROOTS 197
Arithmetic 1.0: Preparing for Algebra 198
Doing Arithmetic with "Real" Numbers 201
Arithmetic 2.0 (Here Comes the Algebra) 214
Algebra Tutorial: How to Simplify Expressions 220
Math Properties 223
Algebra Tutorial: How to Combine Like Terms 238
Exponentiation 240
Roots 248
Exponentiation and Roots Combined 254
Irrational Exponents 256
Can You Remind Me of the Point of Algebra? 259
Final Exam 260

PART III: SOLVING ALGEBRA PROBLEMS

CHAPTER 5: POLYNOMIALS, FUNCTIONS, AND BEYOND 265
What Are Polynomials? 266

Evaluating Polynomials 274
Functions 279
Visualizing Polynomial Functions 287
How to Solve Problems with Polynomials 296
Algebra Tutorial: How to Find and Write the Equations of Lines 300
Equations of Horizontal and Vertical Lines 309
Systems of Equations 320
Algebra Tutorial: How to Solve a System of Equations 321
Systems of Inequalities 328
Algebra Tutorial: How to Solve a System of Inequalities 329
Challenge Problem #1 333
Final Exam 335

CHAPTER 6: THE ROOT OF THE PROBLEM 339
Challenge Problem #2 339
Roots of Polynomials 344
Factoring Polynomials 360
Algebra Tutorial: How to Factor Polynomials 372
Solving Quadratic Equations 383
Challenge Problem #3 401
Game Over? 409
Final Exam 410

The Math Dude's Solutions 413
Acknowledgments 475
Index 477

Introduction

Greetings, math fan!

I'm going to go out on a limb and guess that you're holding this book because you want or need to learn algebra. Or perhaps you just thought the cover looked nice and felt compelled to pick it up. Either way, I'm glad you did. And I think you'll be glad that you did too.

WHAT CAN THIS BOOK DO FOR YOU?

My offer is to teach you not just how to do algebra, but about the meaning of it too. Why should that be important to you? Well, let me give you an analogy. Though it's useful to learn to follow a couple of recipes that will help you prepare a few meals, eventually you'll grow tired of eating the same chicken stir-fry day after day after day. But if you instead learn about ingredients and all the ways you can combine and prepare them, then you'll have learned to cook. And with that knowledge will come

053-47216_ch01_5P.indd 1 5/10/11 4:40 PM

a lifetime of lasagna and steak and risotto and lobster—in other words, you won't be doomed to a life of just chicken stir-fry.

That's exactly analogous to what we're going to do with algebra! When all is said and done, you're not just going to know how to do a couple of specific problems, or cook some classic "recipes"; you're going to understand how and why algebra works so that you have the ability to solve all the problems this book doesn't teach you to solve too—problems that you might see on the SATs, on a final exam, or just on your homework. That's the true test of whether or not you have really learned something. And that's exactly the test that this book is aimed at helping you pass.

WHO CARES ABOUT ALGEBRA?

I certainly do. And I hope that you do too. Maybe you're a middle school or high school student taking an algebra class. Or perhaps you're a (slightly embarrassed) parent of one of those kids that used to help with homework but now hides at the sound of their son's or daughter's approaching footsteps. Or maybe it's been a few years since your last math class and you need to refresh those memories for your new job. And, of course, there are those people who want to learn simply because they know that there's nothing better in life than discovering something new—or rediscovering something old. Maybe you're one of those lucky people.

In truth, it doesn't really matter which of those you are—or whether or not your description matches any of them. Because this book is here for each and every one of you. And so am I. Honestly, if you have questions while reading the book, need help understanding something, or just want to chat about math, you're more than welcome to write me . . . I'd love to hear from you. My e-mail address is mathdude@quickanddirtytips.com.

WHO IS THE MATH DUDE?

When I was young—before I learned algebra or much math at all—I remember thumbing through a dusty and ancient-looking algebra book I found in my great-grandmother's basement. I can remember looking at all the symbols and equations on the pages and thinking that it looked really awesome. I had no idea how it worked or what any of it meant, but I certainly wanted to know. Kind of strange, I know. But that's the kind of kid I was . . . and it's still the kind of person I am today—curious. I'm never satisfied to know only *that* something works because what I really want to know is *why* it works. To me, that's the good part!

Once I got a little older, I realized that the universe is a really big place . . . and that I wanted to know how it works too. So I studied astrophysics at UCLA, and eventually earned a Ph.D. from Cornell University. Which means that my background with math is not, as you might expect, as a mathematician or a math teacher, but is instead as a professional user of math. After a computer, a familiar programming language, and a cup of coffee or two, math is the most essential tool that I have to help me study the universe.

After graduate school, I made a few important discoveries. First, though I had always been good at math and wanted to know as much about it as possible, I must admit that I had never found it beautiful. But at some point curiosity (and the Internet) got the better of me, and I started reading about and playing with math . . . just like I had wanted to do as a kid. And what I discovered was truly exquisite. Oh, at some point in there I also made the discovery that I'm not half bad at explaining complicated things to people.

Which leads me directly to why you're reading these very words at this very moment. Because in January of 2010, I took action on the twin discoveries I'd made and became Quick and Dirty Tips' resident Math Dude. Each and every week since then I've been writing and hosting a new podcast episode of *The Math Dude's Quick and Dirty Tips to Make Math Easier.* iTunes even

named it one of the best new podcasts of 2010! So, if you end up liking what you read here, I encourage you to join the legions of "math fans" and give the podcast a listen. You can find the show on iTunes or at http://mathdude.quickanddirtytips.com.

WHAT'S IN THIS GUIDE TO ALGEBRA?

If you're wondering if this book will cover *all* of algebra, the answer is no. No book can. But it does cover the big important topics that you'll see in a typical first course in algebra—usually called Algebra I. Which means that we will cover everything from linear equations to polynomials to how to solve quadratic equations.

HOW IS THIS GUIDE ORGANIZED?

In order to learn any subject, you have to practice. It's not enough just to read about something and assume that you'll then be able to do it. That's why in each chapter you'll find lots of questions organized into three different types of sections that are designed to help you test your knowledge:

- **Pop Quizzes:** There are several quizzes per chapter designed to give you a chance to test your understanding of a topic right after reading about it. That way, if you have trouble with the quiz, you know that you should go back and review that section before moving on. Some of the quizzes contain new information or interesting applications that you won't find in the text, so they should be considered "required reading."
- **Algebra Tutorials:** Most chapters contain one or more tutorials designed to teach you a particular algebra technique or walk you through the method for solving a certain type of algebra problem. These tutorials encourage your active participation by occasionally leaving a step unfinished so that you can make the final mental connections and fill in

the details or by prompting you with questions to help you test your understanding.

- **Final Exams:** After you've gone through all the Pop Quizzes and algebra tutorials in each chapter, you'll find one final test of your understanding of the material you just learned in the chapter final exam. Many of the problems in these "exams" contain applications of the material in the chapter that we haven't talked about before. So like the quizzes, these exams should be considered "required reading" . . . and required doing for that matter!

But having quizzes, tutorials, and exams with questions to help you test your understanding is really only useful if you can check whether or not you've answered those questions correctly. So, at the end of the book you'll find a complete and thorough solutions guide containing *full explanations* of how to solve all the questions in the book. Which means that if you're stuck on a quiz or final exam question, you can always turn to Solutions and learn how to get yourself out of that jam. It's like having a personal tutor right in your book.

In addition to these sections that are designed to test your knowledge, you'll find several other reoccurring sections that will help you understand algebra better and make it fun. Amazing, I know.

- **Secret Agent Math-Libs:** Each chapter contains a math-lib section that follows a day in the life of a math secret agent. Yes, you heard that right—these sections do indeed take you on an algebra-inspired spy adventure full of codes to crack and mysteries to unravel. And while you should definitely enjoy the puzzles and twists in these sections, you'll also be dealing with real algebra problems . . . many of which will give your brain a real workout and get you thinking about math in a whole new way.
- **Math Brain Game:** Each chapter also contains a Math Brain Game section designed to get you to think about

algebra problems in new and interesting ways . . . often visually. These sections typically give you some background information, then introduce you to the problem, and finally leave you to think about how to solve it. Of course, this is then followed up with a thorough explanation of how to actually do the solving . . . just in case you didn't figure it all out yourself.

- **Overachiever Badge:** When it comes to algebra, there are a ton of interesting things we could talk about. And there are probably two tons of interesting applications we could talk about. But, obviously, we don't have time to cover three tons worth of stuff. So certain topics that are interesting but not absolutely essential are discussed in the Overachiever Badge sections. And if you read all of these sections, you'll earn an Overachiever Badge! Who doesn't want that?
- **Watch Out!:** Every road has its tricky points that can potentially lead to danger: the curve that came out of nowhere, the giant pothole that opened up overnight, the stop sign that you swear wasn't there yesterday. And the road to algebra is no different. But I'll help you avoid those "dangers," aka common mistakes, in the Watch Out! sections.
- **Algebra Decoder!:** There's a lot of "jargon" in algebra. And by that I mean that there are a lot of algebra-specific words and terms that you don't use in everyday conversation. For many of these terms, you'll find Algebra Decoder! sections that will fill in the details about what they mean and where they come from.
- **Quick and Dirty Tips:** Since this is *The Math Dude's Quick and Dirty Guide to Algebra,* and my podcast is called *The Math Dude's Quick and Dirty Tips,* I'd be cheating you if I didn't give you some quick and dirty tips in the book. Which is exactly what you'll find sprinkled throughout the book.

HOW SHOULD YOU USE THIS BOOK?

There are two options for using this book. The first is to just read it . . . and that's no joke. A lot of math books are filled with far more equations than words and include scant explanations. But this book is different. If you thumb through its pages, you'll see lots of words to describe the topics and algebra problems that we're learning to solve . . . and also lots of drawings to help you visualize their meaning and put a tangible spin on an otherwise abstract subject.

The book was written with the reader in mind, and if you do read it from cover to cover and work through the many quizzes, tutorials, and final exams that it contains, you'll come away with a top-of-the-class deep understanding of algebra.

But I know that not everybody in the real world will have time to read the entire book—and that's okay. Yes, it's perfectly fine to look up and read only the sections that you need help on and skip the rest . . . for now, at least. Of particular use to you if you're this type of reader are the tutorials and quizzes. My advice is to first go through the tutorial on the topic that you're studying, and then take a look at the follow-up practice problems and their solutions to help you figure out how to solve the particular problems you're dealing with.

But I hope, in the end, that no matter which type of reader you are, you'll end up wanting to read more . . . because I genuinely think that if you allow yourself to get into and discover algebra for what it really is, you'll like it. And I genuinely hope that you enjoy reading and working though this book as much as I've enjoyed writing it!

So, here's to hoping we spend a lot more time together learning algebra.